STUDIES ON GRANULAR MATERIALS. VI.*

CONTRIBUTION TO THE MEASUREMENT OF NORMAL AND SHEAR STRESSES IN WALLS OF BUNKERS**

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A ring shaped elastic element for the measurement of forces acting on the walls of bunkers with granular materials is described. By means of strain gauges attached to the elastic element it is possible to measure simultaneously the normal and shear stresses. This element is characterised by a simple shape, a small number of strain gauges and a good sensitivity in force measurements of both stresses. The stress distribution in the ring and the mechanical and electrical principles of measurement are studied in detail.

An important factor in the design of steel or reinforced concrete bunkers and silos is the magnitude and distribution of the force acting on the walls, hopper or bottom and the manner in which this force manifests itself. It has been proved that the forces on the walls and bottom do not increase proportionally with the height of granular material. In contrast to liquid behaviour a part of the weight of the stored granular material is transmitted onto the walls by friction. Roscoe and Schofield[1-3] have shown that the measurement of the normal stress without the shear stress gives incomplete data on the stress distribution on the walls of bins and bunkers. Arthur and Roscoe[4] therefore designed a pressure cell for measuring the normal and shear stress as well as the excentricity of the normal force acting on the active face of the pressure cell. Other types of pressure cells measuring normal and shear stresses were designed by Delaplaine[5], Kovtun, Platonov[6] and Broersma[7].

The aim of this paper is to describe in more detail a very simple elastic element[9] which meets the requests for normal and shear stress measurements on bunker walls with a minimum number of strain gauges, gives a reliable time record of the measured forces and makes it possible to transmit the voltage signal to places remote from the measuring element. It consists of a modified mechanical dynamometric ring in combination with strain gauges (Fig. 1). The applied normal and shear forces cause bending stresses in the ring whereby deformations on the outer and inner periphery of the ring are produced. These deformations are combined in pairs

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of equal positive and negative (tension and pressure) values. This corresponds to the stresses exerted on the outer fibres of bent beams on opposite sides of the neutral axis of the perpendicular cross-section of the beam. The sensitivity is in this case amplified owing to bigger relative deformations. In contrast to mechanical dynamometers where only radial forces can be measured the combination of strain gauge measurement of stresses on the dynamometric ring and suitable wiring of the strain gauges allows the measurement of forces having an arbitrary direction. However, the dynamometric ring has to be designed as a thin curved bar ($r/h > 10$) so that in perpendicular cross-sections of the ring we can assume linear distribution of bending stresses as is done in straight bars. In order to be able to fix the strain gauges into places giving the greatest measuring sensitivity, it is necessary to determine the stress distribution in the ring caused by both the normal P_r and shear P_t forces, since on the perimeter of the ring there are locations where the outer fibres have maximum strains and on the other hand there are locations where the strains are small or zero.

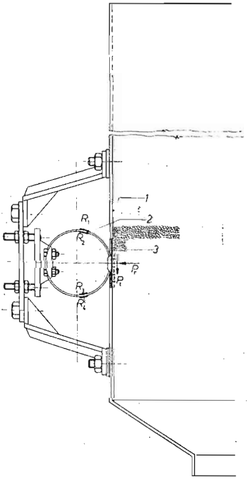


FIG. 1

Cell for Measuring Normal and Shear Stress on Walls of Bunkers

1 Bunker wall, *2* strain gauges, *3* granular material.

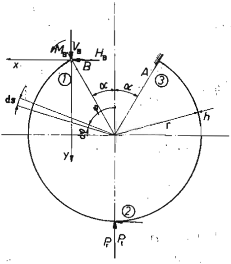


FIG. 2

Calculation Diagram of Ring

Stress Distribution in the Dynamometric Ring

The following calculations of bending moments of the dynamometric ring are based on the validity of Castiglian's[8] theorem. The ring (Fig. 2) is three times statically indeterminate (has three superfluous reactive elements), symmetrical in shape and unsymetrically loaded. Therefore, in the releasing point *B* there will be acting three statically indeterminate quantities – reaction V_B, H_B and couple M_B. To obtain a solution three conditions of deformation are necessary. Zero deformations in point *B* can be expressed by means of the second Castiglian's theorem. The reactions V_B, H_B and M_B can be determined by solving the system of three equations with three unknowns.

The forces acting on the ring give rise to the following bending moments: between points *1* and *2*

$$\mathrm{Mo}_{1-2} = V_B x - H_B y - M_B\,, \quad \text{for} \quad \alpha \leqq \varphi \leqq \pi\,; \tag{1}$$

between points *2* and *3*

$$\mathrm{Mo}_{2-3} = V_B x - H_B y - M_B - P_r(x + r\sin\alpha) + P_t[r(1 + \cos\alpha) - y] \quad \text{for} \quad \pi \leqq \varphi \leqq 2\pi - \alpha\,. \tag{2}$$

Conditions of deformation:

$$\Delta x_B = \frac{\partial L}{\partial H_B} = 0 = \frac{1}{EI}\left[\int_1^2 \mathrm{Mo}_{1-2}\,\frac{\partial \mathrm{Mo}_{1-2}}{\partial H_B}\,\mathrm{d}s + \int_2^3 \mathrm{Mo}_{2-3}\,\frac{\partial \mathrm{Mo}_{2-3}}{\partial H_B}\,\mathrm{d}s\right], \tag{3a}$$

$$\Delta y_B = \frac{\partial L}{\partial V_B} = 0 = \frac{1}{EI}\left[\int_1^2 \mathrm{Mo}_{1-2}\,\frac{\partial \mathrm{Mo}_{1-2}}{\partial V_B}\,\mathrm{d}s + \int_2^3 \mathrm{Mo}_{2-3}\,\frac{\partial \mathrm{Mo}_{2-3}}{\partial V_B}\,\mathrm{d}s\right], \tag{3b}$$

$$\Delta \varphi_B = \frac{\partial L}{\partial M_B} = 0 = \frac{1}{EI}\left[\int_1^2 \mathrm{Mo}_{1-2}\,\frac{\partial \mathrm{Mo}_{1-2}}{\partial M_B}\,\mathrm{d}s + \int_2^3 \mathrm{Mo}_{2-3}\,\frac{\partial \mathrm{Mo}_{2-3}}{\partial M_B}\,\mathrm{d}s\right]. \tag{3c}$$

Equations (*1*) and (*2*) after transformation of coordinates

$$x = r(\sin\varphi - \sin\alpha)\,, \quad y = r(\cos\alpha - \cos\varphi)\,, \quad \mathrm{d}s = r\,\mathrm{d}\varphi\,; \quad r = \text{const.} \tag{4}$$

are substituted into the conditions of deformation (*3*). After integration we obtain a system of equations in the form

$$a_{11}rV_B + a_{12}rH_B + a_{13}M_B = a_{14}rP_r + a_{15}rP_t\,, \tag{5a}$$

$$a_{21}rV_B + a_{22}rH_B + a_{23}M_B = a_{24}rP_r + a_{25}rP_t\,, \tag{5b}$$

$$a_{31}rV_B + a_{32}rH_B + a_{33}M_B = a_{34}rP_r + a_{35}rP_t\,, \tag{5c}$$

where constants a_{ij} are dependent on the included angle α. The values of the constants are

$$a_{11} = 1 + (\pi - \alpha)\sin 2\alpha - \cos 2\alpha,\ a_{12} = 2(\pi - \alpha) + \tfrac{3}{2}\sin 2\alpha + (\pi - \alpha)\cos 2\alpha,$$

$$a_{13} = 2[\sin\alpha + (\pi - \alpha)\cos\alpha],\ a_{14} = \tfrac{1}{4}[3 + \cos 2\alpha + 4\cos\alpha],\ a_{15} = -\tfrac{1}{2}(\pi - \alpha) +$$
$$+ \sin\alpha - \tfrac{1}{4}\sin 2\alpha + (\pi - \alpha)\cos\alpha,\ \ a_{21} = 2(\pi - \alpha) + \tfrac{1}{2}\sin 2\alpha - (\pi - \alpha)\cos 2\alpha,$$

$$a_{22} = 1 + (\pi - \alpha)\sin 2\alpha - \cos 2\alpha,\ a_{23} = 2(\pi - \alpha)\sin\alpha,\ a_{24} = \tfrac{1}{2}(\pi - \alpha) + \sin\alpha +$$
$$+ \tfrac{3}{4}\sin 2\alpha,\ a_{25} = \tfrac{1}{4} + (\pi - \alpha)\sin\alpha + \cos\alpha + \tfrac{3}{4}\cos 2\alpha,\ a_{31} = a_{23},\ a_{32} = a_{13},$$
$$a_{33} = 2(\pi - \alpha),\ a_{34} = 1 + \cos\alpha,\ a_{35} = \pi - \alpha - \sin\alpha.$$

The system of equations (*5a*)–(*5c*) is solved for the unknown V_B, H_B and M_B in the form

$$V_B = a_1 P_r + a_2 P_t\,,\quad H_B = b_1 P_r + b_2 P_t\,,\quad M_B = c_1 r P_r + c_2 r P_t\,. \qquad (6a, b, c)$$

Substituting Eqs (*6*) into Eqs (*1*) and (*2*) and after rearrangement we obtain the final expressions for the distribution of the bending moment in the generalised form

$$\mathrm{Mo}_{1-2} = P_r r(C_1 \sin\varphi + C_2 \cos\varphi + C_3) + + P_t r(C_4 \sin\varphi + C_5 \cos\varphi + C_6)\,, \qquad (7)$$

$$\mathrm{Mo}_{2-3} = P_r r(D_1 \sin\varphi + D_2 \cos\varphi + D_3) + + P_t r(D_4 \sin\varphi + D_5 \cos\varphi + D_6) \qquad (8)$$

and the general expression for the constants C_{1-6} and D_{1-6} are

$$C_1 = a_1,\ D_1 = a_1 - 1,\ C_2 = b_1,\ D_2 = b_1,\ C_3 = -a_1\sin\alpha - b_1\cos\alpha - c_1,\ D_3 =$$
$$= -a_1\sin\alpha - b_1\cos\alpha - c_1,\ C_4 = a_2,\ D_4 = a_2,\ C_5 = b_2,\ D_5 = b_2 + 1,\ C_6 =$$
$$= -a_2\sin\alpha - b_2\cos\alpha - c_2,\ D_6 = -a_2\sin\alpha - b_2\cos\alpha - c_2 + 1.$$

A detailed study[10] containing calculations of the individual constants characterising the location of the strain gauges, a programme for calculating bending stresses in the ring on computer NE 803B and experimental results prove the applicability of the elastic element for the above mentioned purpose. A characteristic distribution of the bending stresses in the ring is shown in Fig. 3*a*, *b*, *c*.

Mechanical Principle of Measurement

Let us consider a dynamometric ring (Fig. 3), fixed in two points, loaded by a general force P which is composed of normal P_r and shear P_t components. At every point of the ring's mean diameter bending stresses are developed under the action of forces P, *i.e.* forces P_r and P_t and these are measured by strain gauges. If we calculate the bending stresses along the mean diameter of the ring, in our case considered to be a thin curved bar, we can, assuming symmetry of shape and support, resolve the effect of forces P_r and P_t by the following consideration.

In two points of the ring which are symmetrical to the axis of symmetry of the circular part of the ring (*i.e.* at points corresponding to angles φ and $2\pi - \varphi$ in Fig. 3) force P_r induces a bending moment of equal size and sign while force P_t causes

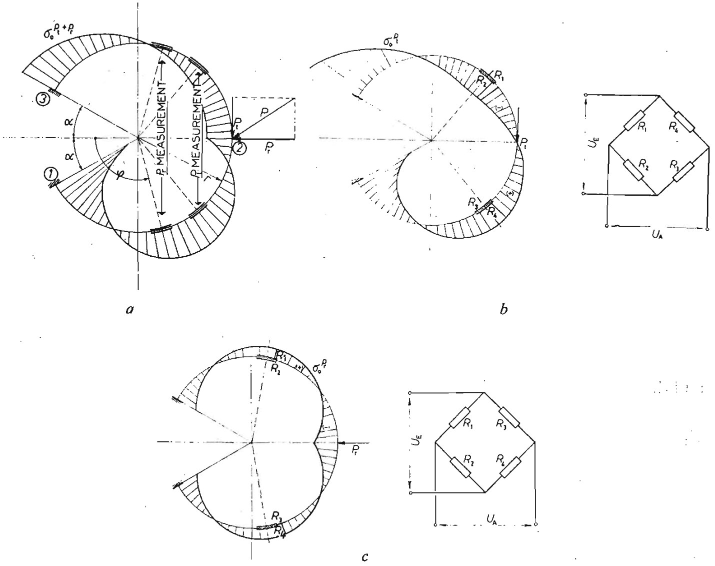


FIG. 3

Diagram of Stress Distribution on Ring from *a*) General Force P and Optimum Location of Strain Gauges *b*) Force P_t, Optimum Location of Strain Gauges and Wiring System I *c*) Force P_r, Optimum Location of Strain Gauges and Wiring System II a bending moment of equal size but opposite sign. If we designate the moments caused by the forces P_r and P_t as Mo^{P_r} and Mo^{P_t} respectively, the total moments given by superposition are

$$Mo_{1-2} = Mo^{P_r} + Mo^{P_t}, \quad Mo_{2-3} = Mo^{P_r} + (-Mo^{P_t}), \qquad (9), (10)$$

where Mo_{1-2} and Mo_{2-3} denote the bending moments between points *1*–*2* and *2*–*3* respectively. By addition or subtraction of Eqs (*9*) and (*10*) we obtain expressions for determining the moments caused by the individual force components

$$Mo^{P_r} = (Mo_{1-2} + Mo_{2-3})/2, \quad Mo^{P_t} = (Mo_{1-2} - Mo_{2-3})/2. \quad (11, 12)$$

Since the relationship between the forces P_r and P_t and the respective bending moments can be expressed by

$$Mo^{P_r} = P_r r f_1(\varphi, \alpha), \quad Mo^{P_t} = P_t r f_2(\varphi, \alpha), \qquad (13, 14)$$

where $f_i(\varphi, \alpha)$ defines the location of the strain gauge on the ring, the magnitudes of the individual force components can be determined from the expressions

$$P_r = m_1(\sigma_{o1-2} + \sigma_{o2-3}), \qquad (15)$$

$$P_t = m_2(\sigma_{o1-2} - \sigma_{o2-3}), \qquad (16)$$

where

$$m_1 = Wo/2rf_1(\varphi, \alpha), \quad m_2 = Wo/2rf_2(\varphi, \alpha).$$

The principle of separation of bending effects from forces P_r and P_t can be solved by a simple wiring system of the strain gauges.

Tensometric Principle of Measurement

The change in resistance of a strain gauge is proportional to the elongation of the object to which it is attached. This can be expressed as

$$\Delta R/R = k(\Delta l/l) = k\varepsilon. \qquad (17)$$

The deformation constant k for the individual strain gauges is known. It depends mainly on the material from which it is made, it is slightly dependent on the dimensions of the strain gauge, type of zig-zag arrangement of electrical resistance wire and batch of resistance material. In wide temperature intervals it also depends on the temperature. For measurements with strain gauges it is usual to use a Wheatstone bridge. The bridge is always symmetrical *i.e.* $R_1 = R_2$ and $R_3 = R_4$. R_1 is or is not equal to R_3 if one or two strain gauges are connected and $R_1 = R_2 = R_3 = R_4$ if four strain gauges are connected to the bridge (Fig. 4).

The sensitivity of the bridge can be determined when the bridge is in equilibrium, when $R_1 = R_2 = R_3 = R_4 = R$. The usual tolerances in the resistances are, for the ensuing considerations, negligible. The bridge can be balanced *e.g.* according to the wiring shown in Fig. 4. The Wheatstone bridge can be considered as a combination of two voltage dividers, as shown in Fig. 5. It can further be assumed that the entrance resistance of the measuring apparatus is infinitely large, *i.e.* $R_A = \infty$ so that the current I_A entering the apparatus is zero. These conditions are practically met by every electronic measuring apparatus. If strain gauge R_1 is elongated, *i.e.* $R_1 \to R + \Delta R$, then from the distribution of voltages between the individual resistances

$$U_A = U_3 - U_2 = U_E[R/2R - R/(2R + \Delta R_1)] ,$$

$$U_A \approx U_E \tfrac{1}{4}\Delta R_1/R , \quad \text{for} \quad \Delta R_1 \ll R ,$$

$$U_A/U_E = \tfrac{1}{4}\Delta R_1/R = \tfrac{1}{4}k\varepsilon_1 . \qquad (18)$$

If all the branches in the bridge are occupied by active strain gauges, then

$$U_A/U_E = \tfrac{1}{4}k(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4) , \qquad (19)$$

where ε_i is the relative elongation of the object in the location of the strain gauge. If we interchange resistances R_3 and R_4 in the wiring of the Wheatstone bridge, we obtain

$$U_A/U_E = \tfrac{1}{4}k(\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4) . \qquad (20)$$

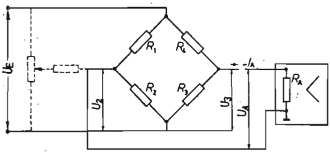


FIG. 4

Wiring of Strain Gauges in Wheatstone Bridge
Dashed lines show connection for balancing bridge.

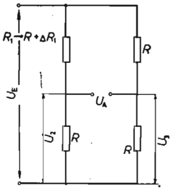


FIG. 5

Voltages on Wheatstone Bridge

By using Eq. (*19*) for the wiring *system I* in Fig. 3*b* we can show that the exit voltage signal from the bridge is proportional only to force P_t, the effect of temperature on the measurement being fully compensated.

$$U_A/U_E = \tfrac{1}{4}k[\varepsilon_1^{P_t} + \varepsilon_1^{P_r} - (-\varepsilon_2^{P_t} - \varepsilon_2^{P_r}) + \varepsilon_3^{P_t} - \varepsilon_3^{P_r} - (-\varepsilon_4^{P_t} + \varepsilon_4^{P_r})], \quad (21)$$

where the upper index indicates the acting force and the sign of ε_i is given by the sense of the stress of the surface fibres in the location of the strain gauge (tension +, pressure −). From the curves of the bending stresses in the ring shown in Fig. 3*b* we can see that

$$\varepsilon_1^{P_r} = \varepsilon_4^{P_r}, \quad \varepsilon_2^{P_r} = \varepsilon_3^{P_r}, \quad |\varepsilon_1^{P_t}| = |\varepsilon_2^{P_t}|, \quad \varepsilon_1^{P_t} = \varepsilon_3^{P_t}, \quad \varepsilon_2^{P_t} = \varepsilon_4^{P_t}.$$

By rearrangement of Eq. (*21*) we have

$$U_A/U_E = \tfrac{1}{4}k(2\varepsilon_1^{P_t} + 2\varepsilon_2^{P_t}) = k\varepsilon^{P_t}. \quad (22)$$

In a similar way we can use Eq. (*20*) for wiring *system II* in Fig. 3*c* where

$$U_A/U_E = \tfrac{1}{4}k[\varepsilon_1^{P_t} + \varepsilon_1^{P_r} - (-\varepsilon_2^{P_t} - \varepsilon_2^{P_r}) - (\varepsilon_3^{P_t} - \varepsilon_3^{P_r}) - \varepsilon_4^{P_t} + \varepsilon_4^{P_r}]. \quad (23)$$

The conditions of the equality of the relative deformations ε_i are the same as in the preceding case except that $|\varepsilon_1^{P_r}| = |\varepsilon_2^{P_r}|$, so that Eq. (*23*) can be simplified to

$$U_A/U_E = \tfrac{1}{4}k(2\varepsilon_1^{P_r} + 2\varepsilon_2^{P_r}) = k\varepsilon^{P_r}. \quad (24)$$

Since the relationships between forces P_t and P_r and the corresponding bending moments are given by Eqs (*13*) and (*14*), we can rearrange Eqs (*22*) and (*24*) to give

$$U_A/U_E = k(\sigma_0^{P_t}/E) = kP_t r f_2(\varphi, \alpha)/E\,\mathrm{Wo} = K_1 P_t,$$

or

$$U_A/U_E = K_2 P_r,$$

where $\sigma_0^{P_t}$ is the stress of a periphery fibre of the ring, $f_2(\varphi, \alpha)$ is a constant for a given ring and position of strain gauge. The constants K_1 and K_2 can be determined from the calibration curves.

By using wiring system I and II it can be seen that the components of the general force P can be separated and that a fourfold amplification with respect to a single strain gauge is obtained. This wiring also ensures perfect temperature compensation. Wiring system I is used for determining the shear components P_t. To obtain maximum sensitivity it is necessary to fix the strain gauge into a location with maximum bending stress. For the measurement of force P_t this location, as is apparent from Fig. 3*b*, is in the vicinity of the place where the dynamometric ring is clamped to the base.

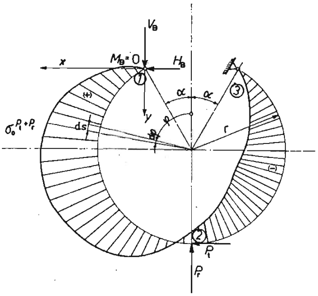
This location, however, is not suitable since the clamping effects can have a negative influence on the measured values of the bending stress. The strain gauges should therefore be fixed nearer to the active force P where there are other two symmetrical and opposite extremes on the stress curves from force P_t. In a similar way wiring system II is used for measuring the normal force component P_r, the shear component P_t being eliminated.

If in attaching the strain gauges we fulfil the condition of symmetricity with respect to force P_r then by the before mentioned method it is possible to measure simultaneously and independently two mutually perpendicular forces P_r and P_t comprising the components of the general force P. Another condition for the reliable measurement is symmetrical fixing of both ends of the elastic element to the base. This can be performed as indicated in Fig. 3*a* or by means of pins. The latter arrangement is also favourable with respect to distribution of bending stresses along the periphery of the ring and in the follow'ng text a shortened solution will be given in the addendum.

ADDENDUM

Let us consider a ring, the ends of which are attached by pins to fixed supports (*i.e.* a two-sided hinged bedding). It is shown that this arrangement of the dynamometric ring is with respect to bending stresses very favourable for the type of measurements we have in mind. However, there exist certain limitations in making a perfect hinged bedding. A solution of the bending stresses in such a ring which is far more simple than in the case of a two-sided clamped ring will now be given. A hinged bedded ring is once statically indeterminate, symmetrical in shape and bedding but unsymmetrically loaded. In the releasing point *1* (Fig. 6) there will be two statically indeterminate quantities — reactions V_B and H_B. To obtain a solution we have to have one static condition of equilibrium and one condition of deformation. Deformations in point *1* are zero, therefore $\Delta x = 0$. The static condition — the sum of moments with respect to point *3* — is zero. In solving these two expressions we obtain the reactions V_B and H_B.

The forces acting on the ring give rise to the following bending moments:



between points *1*—*2*

$$Mo_{1-2} = V_B x - H_B y \,,$$

for $\alpha \leqq \varphi \leqq \pi$ (25)

between points *2*—*3*

$$Mo_{2-3} = V_B x - H_B y - P_r(x + r \sin \alpha) + P_t[r(1 + \cos \alpha) - y] \,,$$

for $\pi \leqq \varphi \leqq 2\pi - \alpha$. (26)

FIG. 6

Calculation Diagram of Ring with Hinged Bedding

The static condition is

$$\mathrm{Mo}_3 = V_B \,.\, 2r \sin \alpha - P_t r(1 + \cos \alpha) - P_r r \sin \alpha = 0 \,. \tag{27}$$

The condition of deformation is

$$\Delta x = \frac{\partial L}{\partial H_B} = 0 = \frac{1}{EI}\left[\int_1^2 \mathrm{Mo}_{1-2} \frac{\partial \mathrm{Mo}_{1-2}}{\partial H_B}\,\mathrm{d}s + \int_2^3 \mathrm{Mo}_{2-3} \frac{\partial \mathrm{Mo}_{2-3}}{\partial H_B}\,\mathrm{d}s\right]. \tag{28}$$

By substituting Eqs (*25*), (*26*) and (*4*) into (*28*) and after integration we obtain the system of Eqs (*27*) and (*28*) in the form

$$a_{11} V_B + a_{12} H_B = a_{14} P_r + a_{15} P_t \,, \tag{29a}$$

$$V_B = a_{24} P_r + a_{25} P_t \,. \tag{29b}$$

Solving for V_B and H_B we get

$$V_B = a_1 P_r + a_2 P_t \,, \quad H_B = b_1 P_r + b_2 P_t \,. \tag{30a, b}$$

where

$a_{11} = 1 + (\pi - \alpha) \sin 2\alpha - \cos 2\alpha$, $a_{12} = 2(\pi - \alpha) + \frac{3}{2} \sin 2\alpha + (\pi - \alpha) \cos 2\alpha$, $a_{14} = \frac{1}{4}(3 + 4 \cos \alpha + \cos 2\alpha)$, $a_{15} = -\frac{1}{2}(\pi - \alpha) + \sin \alpha - \frac{1}{4} \sin 2\alpha + (\pi - \alpha) \cos \alpha$, $a_{24} = \text{const.} = 0{\cdot}500$, $a_{25} = (1 + \cos \alpha)/(2 \sin \alpha)$, $a_1 = a_{24}$, $a_2 = a_{25}$, $b_1 = (a_{14} - a_{24}a_{11})/a_{12}$, $b_2 = (a_{15} - a_{25}a_{11})/a_{12}$.

Substituting Eqs (*30*) into (*25*) and (*26*) we get

$$\mathrm{Mo}_{1-2} = P_r r(C_1 \sin \varphi + C_2 \cos \varphi + C_3) + P_t r(C_4 \sin \varphi + C_5 \cos \varphi + C_6)\,, \tag{31}$$

$$\mathrm{Mo}_{2-3} = P_r r(D_1 \sin \varphi + D_2 \cos \varphi + D_3) + P_t r(D_4 \sin \varphi + D_5 \cos \varphi + D_6)\,, \tag{32}$$

where

$C_1 = a_1$, $D_1 = a_1 - 1$, $C_2 = b_1$, $D_2 = b_1$, $C_3 = -a_1 \sin \alpha - b_1 \cos \alpha$, $D_3 = -a_1 \sin \alpha - b_1 \cos \alpha$, $C_4 = a_2$, $D_4 = a_2$, $C_5 = b_2$, $D_5 = b_2 + 1$, $C_6 = -a_2 \sin \alpha - b_2 \cos \alpha$, $D_6 = -a_2 \sin \alpha - b_2 \cos \alpha + 1$.

From these expressions we can determine the distribution of bending stresses in rings with two-sided hinged bedding. The strain gauges will be attached in locations with maximum values of bending stresses in cross-section of ring.

LIST OF SYMBOLS

a_{ij}	constants in Eqs (*5a*)–(*5c*), where $i = 1, \ldots 3$ and $j = 1, \ldots 5$
a_k	constant in Eqs (*6a*)–(*6c*), where $k = 1, 2$
b_k	constant in Eqs (*6a*)–(*6c*), where $k = 1, 2$
c_k	constant in Eqs (*6a*)–(*6c*), where $k = 1, 2$
$C_{1,\ldots 6}$	constant in Eq. (*7*)
$D_{1,\ldots 6}$	constant in Eq. (*8*)
E	Young's modulus of elasticity (kpm^{-2})

h	height of cross-section of ring (m)
H_B	horizontal statically indeterminate reaction at point B (kp)
I	moment of inertia of cross-section of ring (m^4)
I_A	current of measuring apparatus of bridge (A)
k	deformation constant of strain gauge (—)
$K_{1,2}$	calibration constant ($mVkp^{-1}$)
l	total wire length of strain gauge (m)
Δl	elongation of wire of strain gauge (m)
L	potential energy of state of stress (of deformation) (kpm)
M_B	statically indeterminate moment at point B (kpm)
Mo	bending moment in arbitrary point of ring (kpm)
P	general force (kp)
P_r	normal component of force P (kp)
P_t	shear component of force P (kp)
r	mean radius of ring (m)
R_A	entry resistance of measuring apparatus of bridge (ohm)
$R_{1,\dots 4}$	resistance of strain gauge (ohm)
ΔR	change of resistance due to elongation (ohm)
s	length of perimeter of central line of ring (m)
U_A	exit voltage on bridge (mV)
U_E	entry voltage on bridge (mV)
$U_{1,\dots 4}$	voltage on resistance of strain gauge (mV)
V_B	vertical statically indeterminate reaction at point B (kp)
Wo	section modulus of ring in bending (m^3)
x	length in direction of x coordinate (m)
Δx_B	horizontal displacement (deformation) at point B (m)
y	length in direction of y coordinate (m)
Δy_B	vertical displacement (deformation) at point B (m)
α	included angle of ring
$\varepsilon_{1,\dots 4}$	relative elongation of strain gauge (—)
σ_0	bending stress of ring (kpm^{-2})
φ	angle locating strain gauge on ring (deg)
$\Delta\varphi_B$	variation of angle of tangent of central line of ring at point B (deg)

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